Fourier Transform:-

F.S → Periodic Signals

F.T → non-peradic signals

$$F(jw) = \int_{-\infty}^{\infty} f(t)\, e^{-jwt} \cdot dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(jw)\, e^{jwt} \cdot dw$$

Inuerse Forier Transform:-

Ex:- Find ① F.T of the unit impulse اشتقاق

Signal → $f(t) = \delta(t)$

$$F(w) = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-jwt} \cdot dt$$

$$\int_a^b x(t)\, \delta(t - t_o) \cdot dt$$

$$= x(t_o) \qquad a \le t_o \le b$$

$$F(w) = e^{-jw(0)} = 1$$

① خاصية

③ Frequency shift:-

$x(t) \xrightarrow{F.T} X(\omega)$

$x(t)\, e^{\pm j\omega_0 t} \xrightarrow{F.T} X(\omega \mp \omega_0)$

④ scalling

$x(t) \xrightarrow{F.T} X(\omega)$

$x(at) \xrightarrow{F.T} \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$

⑤ Frequency differntiation:-

$x(t) \xrightarrow{F.T} X(\omega)$

$-jt\,x(t) \xrightarrow{F.T} \frac{d}{d\omega} X(\omega)$

The Amplitude of sinc Function in Frequency domain is equal to the area of the rect in time domain

The angle of the sinc Function equals the ½ of the width of rect Function $\boxed{x\omega}$

Proparties of F.T

① Linearity:-

$x(t) \xrightarrow{F.T} X(\omega)$

$y(t) \xrightarrow{F.T} Y(\omega)$

$x(t)+y(t) \xrightarrow{F.T} X(\omega)+Y(\omega)$

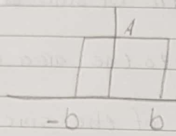
② Time shift:-

$x(t) \xrightarrow{F.T} X(\omega)$

$x(t \pm t_0) \xrightarrow{F.T} X(\omega) \cdot e^{\pm j\omega t_0}$

Ex: اشتقاق

⑤ Find F.T of



$F(t) = A\,\text{rect}\left(\frac{t}{2b}\right)$

$F(w) = A \int_{-b}^{b} e^{-jwt}\,dt = \frac{-A e^{-jwt}}{jw}\Big|_{-b}^{b}$

$F(w) \frac{2}{2} \frac{-A}{jw}\left[e^{-jwb} - e^{jwb}\right]$

$\text{sine}(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

$F(w) = \frac{2}{2}\ \frac{-A}{w}\ \frac{\left[e^{jwb} - e^{-jwb}\right]}{j}$

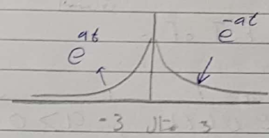
$= \frac{2A}{w} \sin(wb)$ بالضرب وقسم على b

$= \frac{2Ab}{wb} \sin(wb)$

$= 2Ab\ \text{Sinc}(wb)$

اشتقاق

④ Find FT of:-



$f(t) = e^{-a|t|}$

$F(3) = e^{-a|3|} = e^{-3a}$ في حالة $3 = w$, $-3 = w$

$F(-3) = e^{-a|-3|} = e^{-3a}$

$F(w) = \underbrace{\int_{-\infty}^{0} e^{at} \cdot e^{-jwt}\,dt}_{\text{جزء سالب}} + \underbrace{\int_{0}^{\infty} e^{-at} \cdot e^{-jwt}\,dt}_{\text{جزء موجب}}$

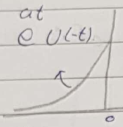
$F(w) = \frac{1}{a+jw} + \frac{1}{a-jw}$ توحيد مقام

$F(w) = \frac{a - \cancel{jw} + a + \cancel{jw}}{a^2 - \boxed{j^2} w^2} = \frac{2a}{a^2 + w^2}$

$j^2 = -1$

اشتقاق

③ Find F.T of :-



$F(t) = e^{at}\,U(-t)$, $a > 0$

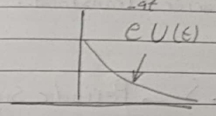
$F(\omega) = \int_{-\infty}^{0} e^{at} \cdot e^{-j\omega t}\, dt$

$F(\omega) = \int_{-\infty}^{0} e^{(a-j\omega)t}\, .dt$

$F(\omega) = \dfrac{e^{(a-j\omega)t}}{a-j\omega}\Bigg|_{-\infty}^{0} = \dfrac{1-0}{a-j\omega} = \dfrac{1}{a-j\omega}$

اشتقاق

② Find Fourier Transform



$F(t) = e^{-ab}\,U(t)$, $a>0$

$F(\omega) = \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t}\, .dt$

$F(\omega) = \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$

$F(\omega) = \dfrac{-e^{-(a+j\omega)t}}{a+j\omega}\Bigg|_{0}^{\infty}$

$F(\omega) = \dfrac{-(0-e^{0})}{a+j\omega} = \dfrac{1}{a+j\omega}$

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6

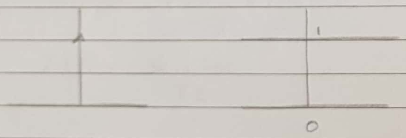
استنتاج

$$\int_{-\infty}^{t} \delta(t)\,.dt = u(t)$$

$$x(t) = \delta(t)$$

$$\delta(t) \xrightarrow{F.T} 1$$

$$\int_{-\infty}^{t} \delta(\tau)\,.d\tau \xrightarrow{F.T} \frac{1}{j\omega}\,.1 + \pi\,.1\,.\delta(\omega)$$



$$u(t) \xrightarrow{F.T} \frac{1}{j\omega} + \pi\,\delta(\omega$$

⑥ Time Diffrentiation:-

$$x(t) \xrightarrow{F.T} X(\omega)$$

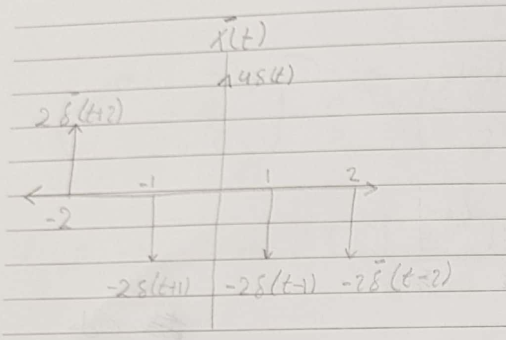
$$\frac{d}{dt}\,x(t) \xrightarrow{F.T} j\omega\;X(\omega)$$

$$\frac{d^n}{dt^n}\,x(t) \xrightarrow{F.T} (j\omega)^n\;X(\omega)$$

⑦ Integration:-

$$x(t) \xrightarrow{F.T} X(\omega)$$

$$\int_{-\infty}^{T} x(\tau)\,.d\tau \qquad \frac{1}{j\omega}\,X(\omega) + \pi X(0)\,\delta(\omega)$$



$$\frac{d^2(x)}{d(t)^2} \; \cancel{\frac{d^2x(t)}{dt}} = 2\underbrace{\delta(t+2)}_{e^{2j\omega}} - 2\underbrace{\delta(t+1)}_{e^{j\omega}} + 4\delta(t) - 2\underbrace{\delta(t-1)}_{e^{-j\omega}} - 2\underbrace{\delta(t-2)}_{e^{-2j\omega}}$$

↓ F.T

$$(j\omega)^2 X(\omega) = 2\boxed{j\omega}e^{2j\omega} - 2e^{j\omega} + 4 - 2e^{-j\omega} - 2\boxed{j\omega}e^{-2j\omega}$$

$$X(\omega) = \frac{2\left[-j\omega e^{2j\omega} + e^{j\omega} + e^{-j\omega} - 2\right]}{\omega^2}$$

ضربت في سالب واحد
لأن j = -1

(12)

$$x(t) = \sin(\omega_0 t)$$

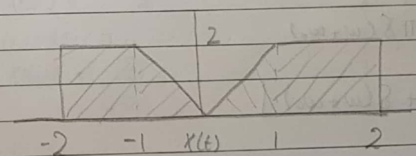
$$\frac{dx(t)}{dt} = \omega_0 \cos(\omega_0 t)$$

$$j\omega X(\omega) = \omega_0 \pi \left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$

$$X(\omega) = \frac{\omega_0 \pi}{j\omega}\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$

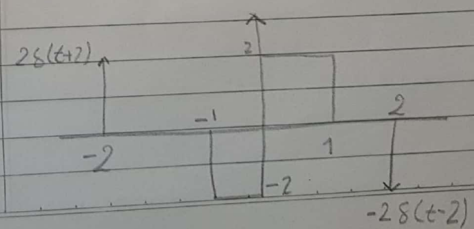
(13)

Find F.T of the Following



↓

$$\frac{dx(t)}{dt}$$



11 Find F. T of

$x(t) = \cos(\omega t)$

$$x(t) = \frac{e^{j\omega t}}{2} + \frac{e^{-j\omega t}}{2}$$

Using Frequency shifting property

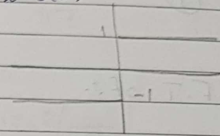
$$X(\omega) = \frac{1}{\cancel{2}}(\cancel{2}\pi\,\delta(\omega-\omega_0)) + \frac{1}{\cancel{2}}(\cancel{2}\pi\,\delta(\omega+\omega_0))$$

$$X(\omega) = \pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)$$

$$= \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$

10 Find F.T of $\quad 1 \rightarrow 2\pi\delta(\omega)$

$x(t) = \mathrm{Sgn}(t)$



$x(t) = 2u(t) - 1$

using time derivative proporty

حل ①

$$\frac{dx(t)}{dt} = 2\,\delta(t)$$

F.T ↓ ↓ F.T

$j\omega X(\omega) = 2$

$$X(\omega) = \frac{2}{j\omega}$$

حل اخر ②

$x(t) = 2u(t) - 1$

$$X(\omega) = 2\left(\frac{1}{j\omega} + \pi\delta(\omega)\right) - 2\pi\delta(\omega)$$

$$X(\omega) = \frac{2}{j\omega} + \cancel{2\pi\delta(\omega)} - \cancel{2\pi\delta(\omega)}$$

$$X(\omega) = \frac{2}{j\omega}$$

Exameple (9)

Find F.T of :-

$x(t) = \frac{1}{9+t^2}$

$\underbrace{e^{-a|t|}}_{x} \xrightarrow{F.T} \frac{2a}{a^2+w^2}$

$w \to t$

$\frac{2a}{a^2+t^2} \xrightarrow[\text{Duality}]{} 2\pi e^{-a|-w|}$

$\frac{2a}{a^2+t^2} \longrightarrow 2\pi e^{-a|w|}$

$a = 3 \rightarrow \frac{6}{9+t^2} \rightarrow 2\pi e^{-3|w|}$ ÷ 6

$\rightarrow \frac{1}{9+t^2} \rightarrow \frac{1}{3}\pi e^{-3|w|}$

whean

$b = 2$

$x(t) = \frac{\sin(2t)}{\pi t}$

$\cancel{4}^{2}A \frac{\sin(2t)}{\cancel{2}t} \xrightarrow{F.T} 2\pi A \operatorname{rect}\left(\frac{w}{4}\right)$ بالقسمة على 2A

Divide both sides by

2A

$\frac{\sin(2t)}{t} \xrightarrow{F.T} \pi \operatorname{rect}\left(\frac{w}{4}\right)$ بالقسمة على π

$\frac{\sin(2t)}{\pi t} \xrightarrow{F.T} \operatorname{rect}\left(\frac{w}{4}\right)$

⑦ Example

Find F.T of 1

$\delta(t) \xrightarrow{F.T} 1$

$1 \xrightarrow{F.T} 2\pi\,\delta(-\omega)$

$\delta(\ )$ is even Function

$1 \xrightarrow{F.T} 2\pi\,\delta(\omega)$

⑧ Example F.T of

$x(t) = \frac{\sin(2t)}{\pi t}$

$A\,\mathrm{rect}\left(\frac{t}{2b}\right) \longrightarrow 2Ab\,\frac{\sin(b\omega)}{b\omega}$

$t \leftarrow \omega$

$\boxed{2Ab\,\frac{\sin(bt)}{bt}} \xrightarrow{F.T} 2\pi A\,\mathrm{rect}\left(\frac{-\omega}{2}\right)$

قانون Duality

لازم يكون نفس شكل

$2\pi\, x(-\omega)$

$A\,\mathrm{rect}\left(\frac{-\omega}{2}\right)$

كل $t \rightarrow -\omega$

$x(t) = \frac{\sin(2t)}{\pi t}$

CAMPAP

⑧ Time Convolation :-

$x(t) \xrightarrow{F.T} X(\omega)$

$y(t) \xrightarrow{F.T} Y(\omega)$

$\underset{\text{convolution}}{x(t) * y(t)} \xrightarrow{F.T} \underset{\text{convolution}}{X(\omega) * Y(\omega)}$

⑨ Duality :- اهم وحدة

$x(t) \xrightarrow{F.T} X(\omega)$

$x(t) \xrightarrow{F.T} 2\pi\, x(-\omega)$

Find inverse F.T of $\frac{1}{(a+jw)^2}$ using convolution property.

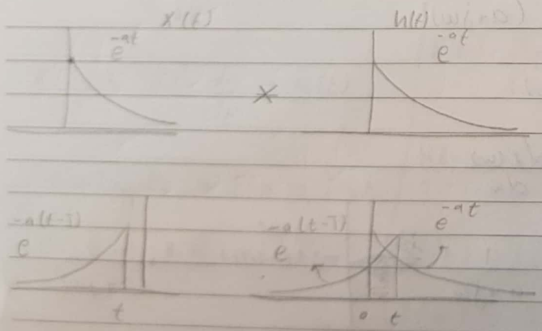
~~Sol~~

$$X(w) = \frac{1}{a+jw} \cdot \frac{1}{a+jw}$$

$$x(t) = e^{-at} \xrightarrow{F.T} \frac{1}{a+jw}$$

Remember that :- $x(t) \overset{\text{convolution}}{*} h(t) \xrightarrow{F.T} X(w) \cdot H(w)$

$$e^{-at}u(t) * e^{-at}u(t) \xrightarrow{F.T} \frac{1}{(a+jw)^2}$$



~~Examples~~

F.T of $\sin(w_0 t)$

$$x(t) = \sin(w_0 t)$$

$$\frac{dx(t)}{dt} = w_0 \cos(w_0 t)$$

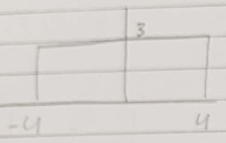
$$jwX(w) = w_0 \left[\pi\delta(w-w_0) + \pi\delta(w+w_0)\right]$$

$$X(w) = \frac{\pi w_0}{jw}\left[\underbrace{\delta(w-w_0)}_{w_0} + \underbrace{\delta(w+w_0)}_{-w_0}\right]$$

$$X(w) = \frac{\pi}{j}\left[\frac{w_0}{w}\delta(w-w_0) + \frac{w_0}{w}\delta(w+w_0)\right]$$

$$X(w) = \frac{\pi}{j}\left[\delta(w-w_0) - \delta(w+w_0)\right]$$

③ $m(t) = 3u(t+4) - 3u(t-4)$



$m(t) = 3\,\text{rect}\left(\frac{t}{8}\right)$

$4 \times 2 \times 3 = 24$

$M(\omega) = \frac{24 \sin(4\omega)}{4\omega}$

④

$r(t) = \frac{1}{1+t^2}$

$e^{-a|t|} \xrightarrow{F.T} \frac{2a}{a^2+\omega^2}$

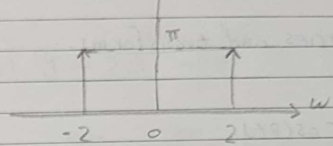
$\frac{2a}{a^2+t^2} \rightarrow 2\pi e^{-a|-\omega|}$

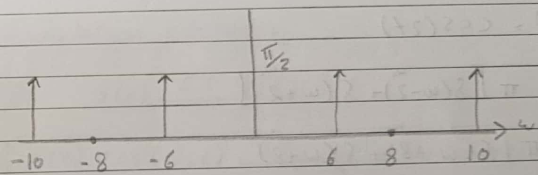
When $a = 1$

$\frac{x(1)}{1+t^2} \rightarrow 2\pi e^{-|\omega|}$

$\frac{1}{1+t^2} \xrightarrow{F.T} \pi e^{-|\omega|}$

$\cos(2t)$





② $f(t) = \text{sgn}(t) + u(-t)$

$$\boxed{\begin{array}{l} \text{sgn}(t) = 2u(t) - 1 \\ \text{or} \\ \text{sgn}(t) = u(t) - u(-t) \end{array}}$$

$f(t) = u(t) - \cancel{u(-t)} + \cancel{u(-t)}$

$f(t) \underset{F.T}{=} u(t) \downarrow F.T$

$F(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$

$u(t) = \pi\delta(\omega) + \frac{1}{j\omega}$

Find Forreer Transform For the Following signals using tabble of properties and transforms:

① $s(t) = \cos(2t)\ \cos(8t)$

$x(t)\cos(\omega_0 t)$

$x(t) = \cos(2t)$

$X(\omega) = \pi\left[\delta(\omega-2) + \delta(\omega+2)\right]$

$X(\omega) = \pi\left[\delta(\omega-8) + \delta(\omega+8)\right]$

Using Modulation property:

$$S(\omega) = \frac{\pi}{2}\left[\delta(\omega-2-8) + \delta(\omega+2-8) + \delta(\omega-2+8) + \delta(\omega+2+8)\right]$$

$$S(\omega) = \frac{\pi}{2}\left[\delta(\omega-10) + \delta(\omega+6) + \delta(\omega-6) + \delta(\omega+10)\right]$$

$r \xrightarrow{\text{تفاضل}} u \to \delta$

$x''(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$

$(j\omega)^2 X(\omega) = 1 - 2e^{-j\omega} + e^{-2j\omega}$

بالقسمة على $(j\omega)^2$

$j^2 \to -1$

$$X(\omega) = \frac{2e^{-j\omega} - e^{-2j\omega} - 1}{\omega^2}$$

* Modulation property

IF $x(t) \xrightarrow{F.T} X(\omega)$

خاصية Modulation

$$\boxed{x(t)\cos(\omega_0 t) \xrightarrow{F.T} \frac{1}{2}\left[X(\omega-\omega_0) + X(\omega+\omega_0)\right]}$$

$$\cos(\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$\frac{x(t)}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) = \frac{x(t)e^{j\omega_0 t} + x(t)e^{-j\omega_0 t}}{2}$$

F.T

$$\frac{1}{2}\left[X(\omega-\omega_0) + X(\omega+\omega_0)\right]$$

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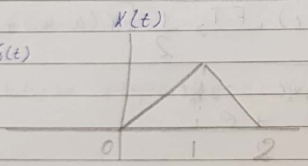
تحويل ناتج حل رقم ①

$$= \left(\frac{e^{j(\omega/2)} - e^{-j\frac{\omega}{2}}}{2j}\right) \cdot \frac{e^{-j\frac{\omega}{2}}}{\omega/2} + \left(\frac{e^{j(\omega/2)} - e^{-j\frac{\omega}{2}}}{2j}\right) \cdot 4 e^{-\frac{3}{2}j\omega}$$

$$= \frac{1 - e^{-j\omega}}{j\omega} + \frac{2\left(e^{-j\omega} - e^{-2j\omega}\right)}{j\omega}$$

$$= \frac{1 + e^{-j\omega} - 2e^{-2j\omega}}{j\omega}$$

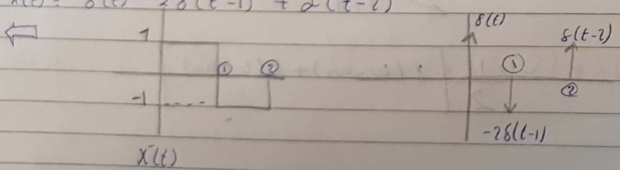
$r(t) \rightarrow u(t) \rightarrow \delta(t)$



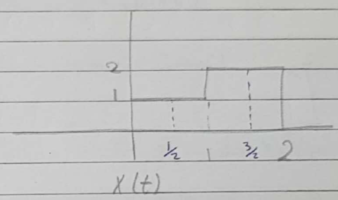
$x(t) = r(t) - 2r(t-1) + r(t-2)$

$x'(t) = u(t) - 2u(t-1) + u(t-2)$

$x''(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$



* Find F.T of:-



حل ①

$x(t) = rect(t - \frac{1}{2}) + 2\, rect(t - \frac{3}{2})$

$$\omega(t) = \frac{\sin(\omega/2)}{\omega/2} e^{-\frac{1}{2}j\omega} + 2\frac{\sin(\omega/2)}{\omega/2} e^{-\frac{3}{2}j\omega} \quad \#$$

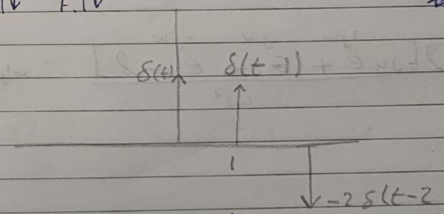
تحويل الى F.T

حل ②

احسن

$u(t) + u(t-1) - 2u(t-2)$

F.T ↓ F.T ↓ F.T ↓



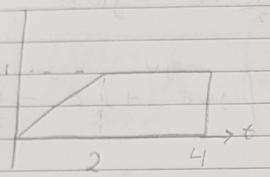
$x(t) = \delta(t) + \delta(t-1) - 2\delta(t-2)$

$j\omega X(t) = 1 + e^{-j\omega} - 2e^{-2j\omega}$

$$X(t) = \frac{1 + e^{-j\omega} - 2e^{-2j\omega}}{j\omega} \quad \#$$

تابع السؤال السابق

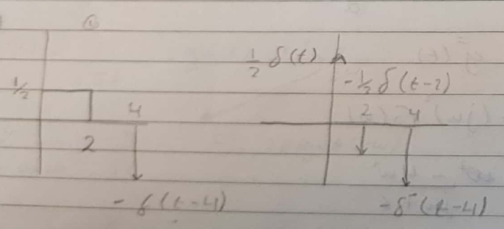
$h(t)$



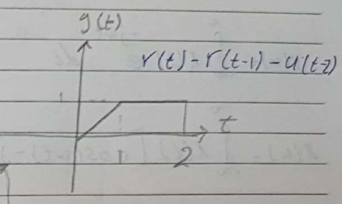
Find $H(\omega)$ if $G(\omega) = \frac{1}{\omega^2}\left[j\omega e^{-2j\omega} + e^{-j\omega} - 1\right]$

$h(t) = g\left(\frac{t}{2}\right)$

For $x(at) \longrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$

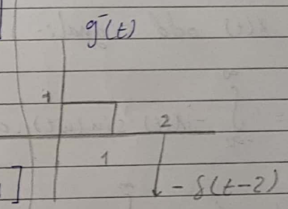


Find F.T of



$(j\omega)^2 G(\omega) = 1 - e^{-j\omega} - j\omega e^{-2j\omega}$

$G(\omega) = \frac{1}{\omega^2}\left[j\omega e^{-2j\omega} + e^{-j\omega}\right]$

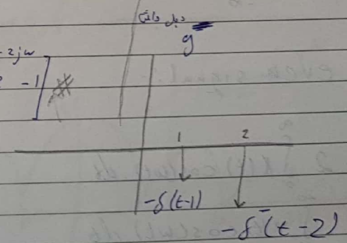


$g(t) \xrightarrow{F.T} \frac{1}{\omega^2}\left[j\omega e^{-2j\omega} + e^{-j\omega} - 1\right]$

$g\left(\frac{t}{2}\right) \xrightarrow{F.T} \frac{2}{\left(\frac{\omega}{2}\right)^2}\left[2j\omega e^{-4j\omega} + e^{-2j\omega} - 1\right]$

$a = \frac{1}{2}$

دبل داش



$h(t) \xrightarrow{F.T} \frac{81}{2\omega^2}\left[2j\omega e^{-4j\omega} + e^{-2j\omega} - 1\right]$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} \, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \left[\cos(\omega t) - j\sin(\omega t)\right] . dt$$

if $x(t)$ odd signal:-

$$X(\omega) = \int_{-\infty}^{\infty} -j x(t) \sin(\omega t) . dt$$

$$X(\omega) = -2j \int_{-\infty}^{\infty} x(t) \sin(\omega t) . dt$$

if $x(t)$ even signal:-

$$X(\omega) = 2\int_{0}^{\infty} x(t) \cos(\omega t) . dt$$

or $= \int_{-\infty}^{\infty} x(t) \cos(\omega t) . dt$

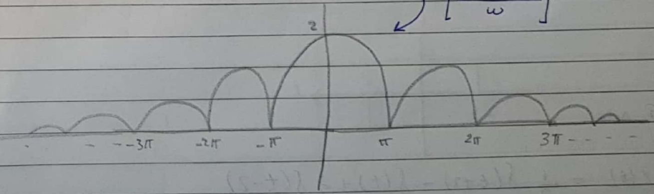
$$(j\omega)^2 X(\omega) = \frac{1}{2} e^{2j\omega} - 1 + \frac{1}{2} e^{-2j\omega}$$

$$X(\omega) = \frac{\cos(2\omega) - 1}{j^2 \omega^2}$$

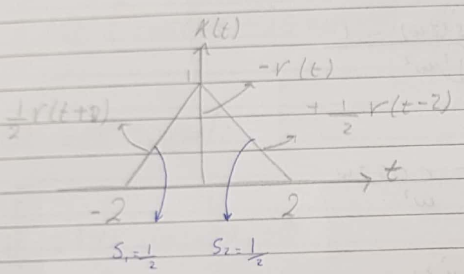
$$X(\omega) = \frac{1 - \cos 2\omega}{\omega^2}$$

$$X(\omega) = \frac{2\sin^2\omega}{\omega^2}$$

$$X(\omega) = 2\left[\frac{\sin(\omega)}{\omega}\right]^2$$ sinc



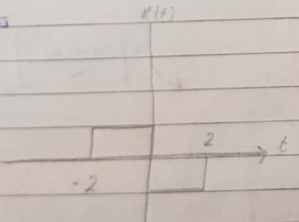
Find F.T of the shown Signal and draw the spectrum



$x(t) = \frac{1}{2} r(t+2) - r(t) + \frac{1}{2} r(t-2)$, تفاضل $\frac{dx(t)}{dt}$

$x(t) = \frac{1}{2} u(t+2) - u(t) + \frac{1}{2} u(t-2)$

$\frac{dx(t)}{dt}$ ① تفاضل



② تفاضل

$$\frac{d^2 x(t)}{dt^2} = \frac{1}{2} \delta(t+2) - \delta(t) + \frac{1}{2} \delta(t-2)$$

$$(jw)^2 X(w) = \frac{1}{2} e^{2jw} - 1 + \frac{1}{2} e^{-2jw}$$

$$Y(w) = 1 \cdot \frac{1}{jwRc+1}$$

$$Y(w) = \frac{1}{jwRc+1}$$

$$e^{-at} u(t) \longrightarrow \frac{1}{a+jw}$$

$$Y(w) = \frac{1}{Rc\left[\frac{1}{Rc} + jw\right]}$$

$$Y(w) = \frac{\frac{1}{Rc}}{\frac{1}{Rc} + jw}$$

For $a = \frac{1}{Rc}$

$$\boxed{\frac{1}{Rc} e^{-\frac{t}{Rc}} u(t)} \xrightarrow{F.T} \frac{1}{Rc} \cdot \frac{1}{\frac{1}{Rc} + jw}$$

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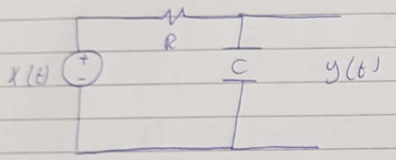
For the shown RC circuit:-

if $x(t) = \delta(t)$. Find the Frequency impulse response and the output



$x(t) = R\,i(t) + y(t)$

$i(t) = C\,\dfrac{dy}{dt}$

$x(t) = RC\,\dfrac{dy}{dt} + y(t)$

$X(\omega) = RC\,j\omega\,Y(\omega) + Y(\omega)$

$X(\omega) = (j\omega RC + 1)\,Y(\omega)$

$x(t) * h(t) = y(t)$

$X(\omega) \cdot H(\omega) = Y(\omega)$

$H(\omega) = \dfrac{Y(\omega)}{X(\omega)}$

$H(\omega) = \dfrac{Y(\omega)}{X(\omega)} = \dfrac{1}{j\omega RC+1}$

$x(t) = \delta(t)$

$X(\omega) = 1$

For $t > 0$

$y(t) = \int_0^t e^{-a\tau} e^{a\tau} e^{-at}\, d\tau$

$y(t) = e^{-at} \int_0^t 1 \,.\, d\tau$

$y(t) = e^{-at} \left[\tau\right]_0^t = t e^{-at}$

$y(t) = t e^{-at} u(t)$

$t e^{-at} \xrightarrow{F.T} \dfrac{1}{(a+j\omega)^2}$

$x(t) \longrightarrow X(\omega)$

$-jt\, x(t) \xrightarrow{F.T} \dfrac{dX(\omega)}{d\omega}$

$e^{-at} u(t) \xrightarrow{F.T} \dfrac{1}{a+j\omega}$

$-jt e^{-at} u(t) \xrightarrow{F.T} \dfrac{-j}{(a+j\omega)^2}$

The input signal of a network is $x(t) = e^{-2t} u(t)$ $\quad \frac{1}{j\omega+a}$

$H(j\omega) = \frac{1}{j\omega+4}$

Find the output $y(t)$

$y(t) = x(t) * h(t)$

$Y(\omega) = X(\omega) . H(\omega)$

$x(t) = \rightarrow X(\omega) = \frac{1}{2+j\omega}$

$Y(\omega) = \frac{1}{j\omega+2} \cdot \frac{1}{j\omega+4}$ كسور جزئية

$Y(\omega) = \frac{A}{j\omega+2} + \frac{B}{j\omega+4}$

① ②

$A = \left.\frac{1}{j\omega+4}\right|_{j\omega=-2} = \frac{1}{2}$

$B = \left.\frac{1}{j\omega+2}\right|_{j\omega=-4} = -\frac{1}{2}$

$Y(\omega) = \frac{1/2}{j\omega+2} - \frac{1/2}{j\omega+4}$

$y(t) = \frac{1}{2} e^{-2t} u(t) - \frac{1}{2} e^{-4t} u(t)$

Fined F.T of:-

$S(t) = \sin(2t + \frac{\pi}{2}) \cos(8t)$

$S(t) = \cos(2t) \cos(8t)$

$\sin(2t + \frac{\pi}{2})$

$= \cancel{\sin(2t) . \cos(\frac{\pi}{2})} + \boxed{\cos(2t)} . \sin(\frac{\pi}{2})$

$= \cos(2t)$

* using Duality Find Fourier transform for:-

$$x(t) = \boxed{\delta(t) + \frac{1}{j\pi t}}$$

ثوابت

$$u(t) \xrightarrow{F.T} \frac{1}{j\omega} + \pi\delta(\omega)$$

$\times 2\pi$

لازم يساوي نفس شكل x(t) في السؤال

$$\boxed{\frac{1}{jt} + \pi\delta(t)} \xrightarrow{F.T} 2\pi u(-\omega)$$

$\div \pi$

$$\frac{1}{j\pi t} + \delta(t) \xrightarrow{F.T} 2u(-\omega)$$

$$f(t) = \frac{1}{j\pi t} \qquad \operatorname{sgn}(t) \xrightarrow{F.T} \frac{2}{j\omega}$$

2π $\div$

تكون نفس الشكل

$$\frac{2}{jt} \xrightarrow{F.T} 2\pi \operatorname{sgn}(-\omega)$$

$$\frac{\cancel{2}}{2\pi jt} \longrightarrow \frac{\cancel{2\pi}}{\cancel{2\pi}} \operatorname{sgn}(-\omega)$$

$$\frac{1}{j\pi t} \xrightarrow{F.T} -\operatorname{sgn}(\omega)$$

$$(j\omega)^2 H(\omega) = \frac{1}{2} \cdot e^{-2j\omega} - j\omega e^{-4j\omega}$$

$$H(\omega) = \frac{1}{\omega^2}\left[j\omega e^{-4j\omega} + \frac{1}{2} e^{-2j\omega} - \frac{1}{2} \right]$$

If:- $G(\omega) = \dfrac{4}{j\omega + 6}$

Find F.T $\{x(t)\}$

where $x(t) = g''(t)$

$x(t) = g''(t)$

$X(\omega) = (j\omega)^2 G(\omega)$

$$X(\omega) = \frac{-4\omega^2}{j\omega + 6}$$